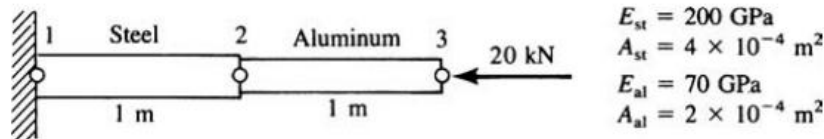1- For the bar assemblages shown in figures, determine the nodal displacements, the forces in each element, and the reactions.

$E_{st} = 200$ GPa
$A_{st} = 4 \times 10^{-4}$ m^2
$E_{al} = 70$ GPa
$A_{al} = 2 \times 10^{-4}$ m^2

Ref: The First Course in the Finite Element, D. L. Logan, 4th Edition.

2- For a bar element, it is proposed to discretize the displacement function as

$$u(x) = N_1(x)u_1 + N_2(x)u_2$$

with interpolation functions

$$N_1(x) = \cos\frac{\pi x}{2L}$$

$$N_2(x) = \sin\frac{\pi x}{2L}$$

Are these valid interpolation functions? (Hint: Consider strain and stress variations.)

Ref: Fundamentals of finite element analysis, D.V. Hutton

3- The torsional element shown in the figure has a solid circular cross section and behaves elastically. The nodal displacements are rotations θ_1 and θ_2 and the associated nodal loads are applied torques T_1 and T_2. Use the potential energy principle to derive the element equations in matrix form.
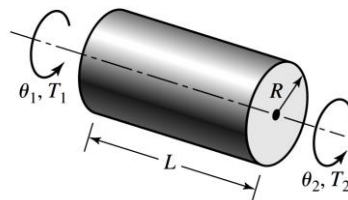
Ref: Fundamentals of finite element analysis, D.V. Hutton

۴- برای المان میله سه‌گرهی نشان‌داده شده در شکل، با کمک قضیه اول کاستیلیانو، ماتریس سختی المان را بدست آورید.